Q1

i) $\frac{d}{dx}$ $\quad 2ye^{x} + e^{x}2\frac{dy}{dx} + 5x^{2}2y\frac{dy}{dx} + y^{2}10x = 0$

$$\frac{dy}{dx}\left(\cancel{2}e^{x} + \overset{5}{\cancel{10}}x^{2}y\right) = -\cancel{2}ye^{x} - \overset{5}{\cancel{10}}xy^{2}$$

$$\boxed{\frac{dy}{dx} = \frac{-ye^{x} - 5xy^{2}}{e^{x} + 5x^{2}y}}$$

ii) $\frac{d}{dx}$ $\quad 3x(\sec^{2}y)\frac{dy}{dx} + (\tan y)3 = 4x$

$$\frac{dy}{dx}\left(3x\sec^{2}y\right) = 4x - 3\tan y$$

$$\boxed{\frac{dy}{dx} = \frac{4x - 3\tan y}{3x\sec^{2}y}}$$

Q2A

a) Sub $y=0$ into eqn.

$$0^{2} + 4x^{2} - e^{0} = 0$$

$$4x^{2} = 1$$

$$x^{2} = \frac{1}{4}$$

$$x = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

Choose the positive value of x.

$$\boxed{x = \frac{1}{2}}$$

Q2b

b) $\frac{d}{dx}$ $\quad 2y\frac{dy}{dx} + 8x - e^{y}\frac{dy}{dx} = 0$

Sub in $x = \frac{1}{2}$, $y=0$

$$\cancel{2(0)\frac{dy}{dx}} + 8\left(\frac{1}{2}\right) - e^{0}\frac{dy}{dx} = 0$$

$$4 - \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = 4}$$

Q3A

a) At $x = 4$

$$2(4)y^2 - 4^2 = 16$$

$$8y^2 = 32$$

$$y^2 = \frac{32}{8} = 4$$

$$y = \sqrt{4} = \pm 2$$ So the points of intersection are: $(4,2)$ $(4,-2)$

$\frac{d}{dx}$ $\quad 2x\,2y\frac{dy}{dx} + y^2 2 - 2x = 0$

$$\left(\frac{dy}{dx}\right)(4xy) = 2x - 2y^2$$

$$\frac{dy}{dx} = \frac{2x - 2y^2}{4xy}$$

At $(4,2)$:

$$\frac{dy}{dx} = \frac{2(4) - 2(2)^2}{4(4)(2)} = \boxed{0}$$

At $(4,-2)$:

$$\frac{dy}{dx} = \frac{2(4) - 2(-2)^2}{4(4)(-2)} = \boxed{0}$$

$\boxed{\therefore \text{ the two gradients are equal.}}$

Q3B

b) Since the gradients are zero at $(4,2)$ and $(4,-2)$, these must be stationary points.

Q4

$x=-1 \quad y=0$

LHS: $3(-1)e^0 + 2(-1) + 5 = -3-2+5 = 0$

RHS: $4(0) = 0$

LHS = RHS

$\therefore (-1,0)$ lies on curve

$y - \underset{\substack{\downarrow \\ 0}}{y_1} = m(x - \underset{\substack{\downarrow \\ -1}}{x_1})$

$\frac{d}{dx} \quad 3x(e^y)\frac{dy}{dx} + e^y 3 + 2 = 4\frac{dy}{dx}$

Sub in $x=-1$, $y=0$

$3(-1)e^0\frac{dy}{dx} + e^0 3 + 2 = 4\frac{dy}{dx}$

$5 = 7\frac{dy}{dx}$

$m = \frac{dy}{dx} = \frac{5}{7}$

$y - 0 = \frac{5}{7}(x-(-1))$

$7y = 5x+5$

$7y - 5x - 5 = 0$

Q5a

a) $\frac{1}{y}\frac{dy}{dx} - \left(2x\ 3y^2\frac{dy}{dx} + y^3 2\right) = 0$

$$\frac{dy}{dx}\left(\frac{1}{y} - 6xy^2\right) = 2y^3$$

$$\frac{dy}{dx} = \frac{2y^3}{\frac{1}{y} - 6xy^2} \times \frac{y}{y}$$

$$\boxed{\frac{dy}{dx} = \frac{2y^4}{1 - 6xy^3}}$$

Q5b

b) $y - y_1 = m_n(x - x_1)$

$y = 1$

$\cancel{\ln(1)} - 2x1^3 = 8$

$x = \frac{8}{-2} = -4$ $(-4, 1)$

$$m_t = \frac{dy}{dx} = \frac{2(1)^4}{1 - 6(-4)(1^3)} = \frac{2}{25}$$

$$m_n = \frac{1}{\frac{dy}{dx}} = \frac{-1}{\frac{2}{25}} = \frac{-25}{2}$$

$$y - 1 = -\frac{25}{2}(x - (-4))$$

$$2y - 2 = -25x - 100$$

$$\boxed{25x + 2y + 98 = 0}$$

Q6

$\frac{d}{dx}$ $x\,2y\frac{dy}{dx} + y^2 - 8x = 0$

At stationary points, $\frac{dy}{dx} = 0$

$\therefore\ y^2 - 8x = 0$

$y^2 = 8x$

Sub in $y^2 = 8x$ into eqn for curve

$x(8x) - 4x^2 = 64$

$4x^2 = 64$

$x^2 = 16$

$x = \pm 4$

When $x = 4$ $\quad y^2 = 8(4) \quad y = \pm 4\sqrt{2}$

When $x = -4$ $\quad y^2 = 8(-4) \quad y = \sqrt{-32}$ no real solutions!

$\therefore$ Stationary points: $\boxed{(4, 4\sqrt{2}),\ (4, -4\sqrt{2})}$

Q7a

a) $LHS = \ln(1 \times 1) + 1 \times 1^2 = 1$

$RHS = 1$

$LHS = RHS$

$\therefore$ $A(1, 1)$ lies on the curve.

Q7B

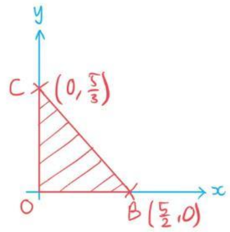
(a) Verify that the point $A(1, 1)$ lies on the curve with equation

$$\boxed{\ln(xy)} + xy^2 = 1$$

$\ln x + \ln y$

[1]

(b) The tangent at point A intercepts the x-axis at point B and the y-axis at point C. Find the area of the triangle OBC.



b) $y - y_1 = m(x - x_1)$

Find gradient, m, of tangent

$\frac{d}{dx}$ $\quad \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + x\,2y\frac{dy}{dx} + y^2 = 0$

$\frac{1}{1} + \frac{1}{1}\frac{dy}{dx} + 1\times2\times1\times\frac{dy}{dx} + 1^2 = 0$

$3\frac{dy}{dx} + 2 = 0$

$\frac{dy}{dx} = -\frac{2}{3} = m$

Find eqn of tangent using

$y_1 = 1, \; x_1 = 1$ and $m = -\frac{2}{3}$

$y - 1 = -\frac{2}{3}(x - 1) = -\frac{2}{3}x + \frac{2}{3}$

Tangent eqn: $y = -\frac{2}{3}x + \frac{5}{3}$

At B, $y = 0$

$0 = -\frac{2}{3}x + \frac{5}{3}$

$x = \frac{5}{2}$

At C, $x = 0$

$y = -\frac{2}{3}(0) + \frac{5}{3}$

$y = \frac{5}{3}$

Area OBC $= \frac{1}{2}bh = \frac{1}{2}\left(\frac{5}{2}\right)\left(\frac{5}{3}\right) = \boxed{\frac{25}{12}}$ square units

Q8

Let $y = a^{kx}$

$\ln y = \ln a^{kx}$

$\ln y = kx \ln a$

$\frac{1}{y}\frac{dy}{dx} = k \ln a$

$\frac{dy}{dx} = yk\ln a$

Sub in $y = a^{kx}$

$$\boxed{\frac{d(a^{kx})}{dx} = ka^{kx}\ln a}$$